Temporal information in Qualitative Simulation

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Abstract

Qualitative simulation is a valuable method for predicting the behavior of partially known dynamical systems. This paper presents an Order of Magnitude Reasoning method which combines purely qualitative simulation technique with some partial numerical information in order to capture temporal information. Duration evaluation methods used so far are based on Mean Value Theorem. However, for critical points, this method is no longer suitable and a method based on the second order Taylor formula is proposed and presented with a validating example.

1. Introduction

Simulation aims at predicting future behavior of a system. Qualitative simulation bases the prediction process on simple reasoning and qualitative calculus rather than on computer power. Qualitative simulation algorithms have shown promising results on a variety of small and moderate sized examples.

Furthermore, numerical models of systems are not always available. This is particularly true for supervision problems (including tasks like diagnosis, monitoring, control ...) in which systems are often complex industrial plants.

In this framework, qualitative simulation can replace numerical simulation to provide qualitative informations, which may be even more appropriate for this kind of tasks.

Kuipers' algorithm QSIM [6] was an essential contribution to the field. This algorithm, based on algebra of signs is presented in Section 2. Although numerous improvements have been added to the first version, some problems still remain as for example obtaining and using adequate temporal information. This problem has been approached in [9] and [15] but no answer was provided at the neighbourhood of critical points.

This paper proposes a method including Order of Magnitude representation and based on second order Taylor formula as a solution for this problem.

After a recall of QSIM principles, orders of magnitude representation is considered to approach problems like how to evaluate durations of simulated phenomenons specially when a critical point is reached. A simple validation example is presented with promising results.

2. The QSIM approach

Qualitative Physics considers that the domain of values representing a physical parameter can be partitionned into a small set of intervals which represent significant qualitative distinctions.

The QSIM algorithm [6] can be considered today as a standard in the qualitative simulation area.

The first step in qualitative modeling is to identify significant variables from the physical point of view. Variables are considered as continuously differentiable functions of time. Then one have to precise some particular values called *landmarks* which delimit several regions.

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Uncompletely known values are described qualitatively in terms of their relation with this discrete set of *landmark values*. The total ordering set of all possible qualitative values a variable can take is called his *quantity space*. Moreover, a variable is not only described by its qualitative magnitude but also by its direction of change, which can be either decreasing, steady or increasing. This pair of values constitutes its *qualitative value*. The set of qualitative values of all the variables of the system represents its *qualitative state*.

A dynamical system model is given by some qualitative relations between variables, called *constraints*. The constraints express some arithmetic relations like $ADD(x,y,z)$, $MULT(x,y,z)$, the differential relation $DERIV(x,y)$ or functionnal relations $M^{+}(x,y)$ and $M^{-}(x,y)$ which specify that x and y have the same or opposite direction of change, respectively. Moreover additionnal information can be given by a tuple of landmark values for which the constraint is satisfied. This tuple is called *corresponding values*.

QSIM proceeds first by generating all the possible qualitative states from the current one. This is made by determining for each variable all the possible transitions. These transitions can be a variable reaching or moving from a landmark, or changing the direction of change.

Then the generated qualitative states which are not compatible with the set of constraints are *filtered* out. QSIM repplies this process to every newly created state and the result is a tree providing all possible behavior.

3. Problems and limitations

Unfortunately, in many realistic situations, even in simple cases, a qualitative description is consistent with an untractible large number of behavioral predictions. And this proliferation increases with the system complexity.

For example for a second order oscillatory system, when the second extremum is reached, this extremum cannot be compared with the first one and three cases are predicted : the new extremum can be bigger, smaller or equal to the previous one. This proliferation of predicted behaviors is one of the main limitations in qualitative simulation today.

Besides, few results exist towards handling duration evaluation in qualitative simulation [8] [9] [15]. Considering for example a tank which empties because of a breakdown, it could be relevant to know if the tank empties in 1h, 24h or 1 month to define an action strategy.

In QSIM the time is represented by symbolic values, which correspond to time points for which a variable reaches a landmark value or an extremum. But there is no way to transpose these symbolic values in numerical ones. This means that a qualitative behavior is only described in terms of a sequence of qualitative states regardless of the duration of each state. Existing methods, with partial numerical information [8] [15] lead more often than not to undetermined duration (one infinite boundary) (see Section 5.3).

Some of the above discussed problems have motivated many researches. Kuipers himself is aware of the existing limitations of his algorithm. He wrote, about the proliferation problem : "the underlying problem is the combination of locality with qualitative description" [6].

Simulation is essentially a local process in the sense that a new state is calculated from information on the preceeding one. It explicitly uses no global information. But in qualitative simulation, information is quite poor. The only knowledge is that a parameter takes his value between two adjacent landmark values which can even be zero and infinite, and that it is increasing, steady or decreasing. It is therefore natural that poor information leads to poor prediction. By making explicit some global properties of the system, this problem can be partially overcame as discussed below.

For example, for the second order oscillatory system some authors [9] have proposed to add an energy constraint in the qualitative model. In this case, one and only one behavior is predicted. In fact, energy constraint is implicitly included in the differential equations. In a numerical simulation, this is expresses by a strict relation between the values of speed and position variables (with truncature errors). But this strict, numerical relation is lost with a qualitative description.

Obtaining new methods to filter out spurious predicted states is a major part of ongoing research in the qualitative simulation area. Recent works include the following topics:

- Methods for changing the level of description in order to aggregate large sets of behaviors whose distinctions are not qualitatively significant [7].

- Methods for reasoning with "higher-order derivative" which provide a curvature constraint to filter out unconsistant states [3] [7].

- More general application of geometric phase space concepts and Lyapunov (generalised "energy") functions to a larger class of second order qualitative differential equations [10] [16] .

Another approach has been developped by Kuipers for decomposing a complex process operating on a widely separated time scales into several simple processes, each at its own time scale [7].

In the first version of QSIM, qualitative reasoning methods were applied only to pure qualitative descriptions in which qualitative values were specified only with ordinal relation with the other values. Kuipers wrote about this : "Ordinal relation are a major part of human commonsense knowledge about quantities, and it is remarkable how many usefull conclusion can be drawn from such an apparently weak description." [8]

Nevertheless it is obvious that human's reasoning considers also proportionality knowledge. This remark leads to the idea of representing *orders of magnitude*. Moreover this provides a way to obtain temporal information about the duration of the phenomenons [2] [9] [15].

4. Orders of Magnitude Representation

4.1. Relative Orders of Magnitude

A first interesting attempt has been made by Ernest Davist [2]. It is based on a mathematical formalism called relative orders of magnitude which uses "non standard numbers algebra". A quantity can be, either a *STANDARD* value, or *SMALL* compared to standard values, or *LARGE*. The qualitative model is, like in QSIM, an abstraction of differential equations.

On a damped spring system, Davist showed that, if the oscillating block is heavy (mass is *LARGE*), then, before the block can reach the zero point, it must pass through a state of *LARGE* duration. Nevertheless, it turns out that this method might be unsufficient. This interesting result applies indeed to a particular case. For example with a *SMALL* or a *STANDARD* block, no temporal conclusion can be provided.

On the other hand, physical interpretation of non standard reals does not seem to be well suited to knowledge representation for simulation purposes. Indeed human representation of magnitudes is essentially a knowledge of proportion. This knowledge vanishes in non standard analysis. Another problem is that using observations or inputs directly from a physical system to be put in the model leads to a difficult numeric/symbolic interface problem.

For all hese reasons,it is our believe that absolute orders of magnitude are a more appropriate representation to be implemented in a qualitative simulation algorithm.

4.2. Absolute Orders of Magnitude

Each landmark is represented by an interval of numerical values which specify the range in which the real value of the landmark may be included. General mathematical properties of such representation have been studied in [19]. The qualitative value of a variable contains therefore partial numerical information. This representation is also used in [9].

Techniques of qualitative calculus presented in [12] can be used to filter some spurious states on *ADD* and *MULT* constraints.

Let the variable $x(t)$ be such that $l_j < x(t) < l_{j+1}$, in the time interval $]t_i, t_{i+1}[$. The qualitative value

of x will be written $x(t) \approx]l_j, l_{j+1}[$ (qualitative equality).
The qualitative equality is defined as follows : let A, B two real intervals, then

$$A \approx B \text{ if and only if } A \cap B \neq \emptyset.$$

Here l_j and l_{j+1} represent themselves intervals. Therefore the interval $]l_j, l_{j+1}[$ represents the interval between the lower boundary of l_j and the upper boundary of l_{j+1}.

No specific notation for qualitative values has been used. $x(t)$ may be a qualitative value or a real one. Ambiguities are discarded vanishes when using qualitative equality. For example $ADD(x,y,z)$ can be written $x+y \approx z$.

4.3. Refinment of M+ functions by orders of magnitude

$M^+(x,y)$ and $M^-(x,y)$ constraints only check whether the directions of change of x and y are equal. These constraints are not well suited to orders of magnitude knowledge propagation. Kuipers' Q2 algorithm [8] uses two numerical enveloppe curves to constraint the relation between x and y. We prefer a modelisation by qualitative piecewise functions. The aim is to obtain an order of magnitude of the slope of the curve $y(x)$.

Modelling a constraint $y=f(x)$ is performed in the following way :

1. Devide the curve in regions of slowly varying slope.
2. Identify pairs of corresponding values to delimit each of these regions.
3. Specify two limit values $p1$, $p2$ for each region of the curve which define the qualitative slope $p =]p1, p2[$ in a given region.

The new constraint is then specified as follows :

$Func\ (x,y),\ ]p_1,\ p'_1[,\ (x_1,y_1),\ ]p_2,\ p'_2[,\ (x_2,\ y_2), \ldots,\ ]p_n,\ p'_n[$

This finer modelisation seems more suitable than M^+ or M^- constraints to cope with available applications knowledge. This is the case for the linear ratio between pressure and height in a tank.

When no additionnal information other than M^+ (or M^-) is available, $Func(x,y)$ still can use the qualitative slope $p =]0,+\infty[$ (or $p =]-\infty,0[$). This remains in accordance with the original idea of qualitative simulation to be able to handle poor knowledge models.

The filtering related to the $Func$ constraint is carried out with order of magnitude qualitative calculus [20]. $Func(x,y)$ is equivalent to $y \approx p.x$. It can be noticed however that when p has one limit value equal to zero, filtering may be unefficient. Research is going on in order to introduce a curvature constraint in such cases.

5. Duration evaluation and temporal filtering

5.1. Generalities

In standard QSIM constraints based qualitative model, time appears (implicitly) through the derivative constraint only. Only the derivative constraint might therefore lead to time evaluation.

Either in QSIM which uses signs algebra, or in Davist' algorithm working with non standard reals, the filtering method is based on the propagation of *order relations* with reference to the corresponding values, which represent "fixed points" for the variables involved in the constraint. Temporal information cannot be provided this way since the time variable is treated differently than other variables and is not referred to landmarks by corresponding values. Therefore, expressing durations needs to restore a numerical ratio between time and other variables, which can be performed through the derivative constraint by using orders of magnitude.

Without mathematical integration of differential equations, there is no way to give a mathematical expression of time as a function of other parameters of the system. Classical formulas of functionnal analysis (Taylor) are therefore necessary to make time explicit. Notice that numerical simulation schemes are based on the same principle.

5.2. Duration evaluation

Let $x(t)$ a variable of the system, that is a continuously differentiable function of time, and consider two time points t_1 and t_2, then, the first order Taylor-Lagrange formula shows that there exist t between t_1 and t_2 such that :

$$x(t_2) = x(t_1) + (t_2-t_1).\dot{x}(t)$$

Therefore, when knowing the values of $x(t_1)$ and $x(t_2)$, two limit values of the derivative leads to two limit values of the duration. Notice that this is not an approximation (as in Taylor-MacLaurin formula) ; all the possible values of the derivative are taken into account to provide all the possible values of the duration. This is in accordance with qualitative simulation philosophy.

The duration corresponding to the state in which the variable x evolves between two consecutive landmarks x_1 and x_2 is evaluated by the following formula :

$$\Delta t \approx (x_2-x_1) / \dot{x}$$

where $\dot{x}$ represents the qualitative value of the derivative between t_1 and t_2, $\approx$ is the qualitative equality [18] and Δt the qualitative value of the duration.

Obviously, the more important the variation of the derivative in the interval $]t_1,t_2[$, the less precise the duration.

Although the formalism is a little different the principle of the calculus is the same as in [8] [15]. It is based on the Mean Value Theorem.

If x is not an original system variable, its expression may be derived analytically from the set of constraints. Kuipers shows a systematic method for obtaining signs of the second order derivatives of all the variables from the higher order derivative of the system [7]. This method is also well suited to obtain orders of magnitude of the first order derivatives.

When evaluating a duration for a qualitative state of x between t_i and t_k, we must be aware that other variables than x may have changed their qualitative value between the time points t_i and t_k inducing some possible changes on the qualitative value of $\dot{x}$. In such cases, the qualitative values of each of those variables on $]t_i,t_k[$ are taken as the union of their qualitative values on the intervals between t_i and t_k. This solution is the simplest but the roughest. Indeed, if the variation of $\dot{x}$ is too important, the evaluated duration will be very unprecise. A solution might be to generate cleverly new landmarks on x or $\dot{x}$ which could delimit intervals on which $\dot{x}$ has weak variation. The total duration could then be evaluated more precisely by evaluating the durations on each of those intervals then addying up. This is one of our research directions at present time.

5.3. Duration evaluation at critical points

Now, consider the case of a critical point for which $\dot{x}$ reaches 0 when x is between x_1 and x_2. QSIM generates automatically a new landmark x^* at a time point t^*. Duration calculus is then unefficient with first order formula. Indeed, zero derivative leads to one infinite boundary for the duration. As time points often correspond to new landmarks creation, this situation occurs often in qualitative simulation and has motivated our work. It is an attempt to overcome the problem of infinite duration.

By using the same procedure as for first order derivative, we can obtain the analytical expression of second order derivatives and use second order Taylor-Lagrange formula: between two time points t, t_0 there exists t' such that :

$$x(t) = x(t_0) + (t-t_0).\dot{x}(t_0) + 1/2.(t-t_0)^2.\ddot{x}(t')$$

If t_0 is choosen as the critical point t*, we get:

$$x(t) = x^* + 1/2.(t-t^*).\ddot{x}(t')$$

Two cases have to be considered :

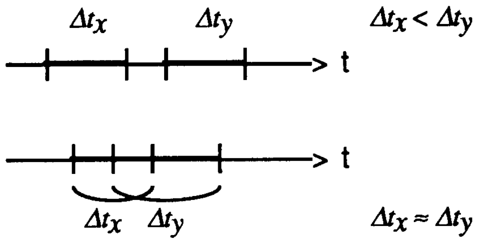
$$t < t^* : \Delta x \approx -1/2.\Delta t^2.\ddot{x}$$
$$t > t^* : \Delta x \approx +1/2.\Delta t^2.\ddot{x}$$

There are two unknown variables : Δx, Δt which contain the undetermined qualitative values x^* and t^*. A second equation is given by the first order derivative constraint on $\dot{x}$:

$$\Delta\dot{x} \approx \Delta t.\,\ddot{x}$$

Except for second order critical points ($\dot{x}$ and $\ddot{x}$ simultaneously equal to zero), an evaluation of Δt and also of x^*, the extremum of x at critical time point t^*, can be provided. Some difficulties of the method are discussed in the example (Section 5.4).

As in [8] [15], the method for temporal filtering is based on the following principle : given the qualitative state duration Δt_x for each variable x which could change state on the next time point, temporal filtering consists in eliminating all the transitions on the variables y for which there exists x, such that $\Delta t_x < \Delta t_y$.



In the second case, it is impossible to filter.

5.4. Examples

In the classical example of the ball throw up with an initial velocity, time evaluated with our method matches exactly theoretical time obtained by integrating differential equation, since it is a second order polynomial, like our Taylor formula. In this example, temporal filtering provides in any case the relative position of the maximum compared to any reference point, which is not obtained by the classical approach.

Consider, now the example of second order oscillating system (spring and a mass without friction).
Let the set of constraints given by :

deriv(x,v).
deriv(v,a).
func(a,x),(0,0),-1.00.

Notice that this is the qualitative model of function $x = sin(t)$. The qualitative slope of the *func* constraint could be choosen as an interval but the aim is to valid the method without propagating any additionnal unaccuracy. The same is true for the initial values given below.

When starting from the initial point:

$t_0 = 1.05s\ (\pi/3)$
$x_0 = 0.87m\ (\sqrt{3}/2)$
$v_0 = 0.5m/s$

At the first time point t_1, a critical point for x $(v_1=0)$ is obtained and the duration $\Delta t = t_1 - t_0$ is such that :

$$x_1 - x_0 \approx 1/2.\Delta t^2.]x_0, x_1[$$
$$v_0 \approx \Delta t.]x_0, x_1[$$

The unknown variables are Δt *and* x_1. The system resolution is performed as follows:

$$\Delta t \approx v_0 /]x_0, x_1[$$
$$(x_1 - x_0).]x_0, x_1[^2 \approx 1/2.v_0^2]x_0, x_1[$$

Then

$$](x_1 - x_0)x_0^2, (x_1 - x_0)x_1^2[$$
$$\approx]1/2.v_0^2.x_0,\ 1/2.v_0^2.x_1[$$

Extremal conditions on $x_1 =]\underline{x_1}, \overline{x_1}[$ have to be deduced from this qualitative equation.
$A \approx B$ means that $A \cap B \neq \emptyset$ i.e. ($\underline{A} < \overline{B}$ and $\overline{A} > \underline{B}$).

Therefore:

$$(\overline{x_1} - x_0)x_0^2 = 1/2.v_0^2.\overline{x_1}$$
$$(\underline{x_1} - x_0)\underline{x_1}^2 = 1/2.v_0^2.x_0$$

We obtain:

$$\overline{x_1} = x_0^3 / (x_0^2 - 1/2 v_0^2) = 1.04$$
$$\underline{x_1}^3 - x_0\,\underline{x_1}^2 - 1/2.v_0^2.x_0 = 0$$
$$\Rightarrow \underline{x_1} = 0.98$$

Then:

$$x_1 =]0.98, 1.04[$$

(The theoretical value calculated from $x = sin(t)$ is 1.00)

We get also:

$$\Delta t \approx 0.5 /]x_0, x_1[= 0.5 /]0.87, 1.04[$$
$$=]0.48, 0.57[$$

(Theoretical value : *0.52*)

The major problem of the method lies on the fact that the expression of x contains generally the unknown variable x_1 what implies algebraic manipulations which require the use of formal calculus system. Relation between $\ddot{x}$ et x_1 depends on the set of constraints.

Another problem appears when starting with the initial state $x_0 = 0$. Indeed between t_0 and t_1 the qualitative state of the system is :

$$x \approx]0, x_1[, inc$$
$$v \approx]0, v_0[, dec$$
$$a \approx]-x_1, 0[, dec$$

We are here at the neighbourhood of two critical points : for x at t_1, for v at t_0. The problem cannot be solved as previously. By considering an arbitrary tuple of corresponding values (x'_0, v'_0, a'_0) at some time point t'_0 $(t_0 < t'_0 < t_1)$, the original time interval is divided into two subintervals containing each only one critical value. Then our method is suitable. By giving a value to one parameter in the tuple, the two others will be deduced by the simulation, as for extremum evaluation. Here, because x_1 is unknown, we must give a value to v'_0. And our interest is to choose v'_0 not too far from v_0 because duration precision is conditionned by the ratio v_0 / v'_0 .

6. Conclusion

This new aproach provides a way to integrate time representation within qualitative simulation. It is an important issue in the field of supervised control system.

Although techniques presented here refer to independant works on order of magnitude calculus, the simulation is based on QSIM principles, that is generating all the possible following states from a current one and then testing the compatibility with the constraints in the model. It is shown that temporal evaluation and filtering can advantageously take part in the simulation process. Systematic procedures to overcome or manage the formal calculus, required by the method, for duration calculus at critical points still have to be developped and implemented.

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